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NUMERICAL ANALYSIS OF GAS FLOW TAKING INTO ACCOUNT RESISTANCE FORCES

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1. To describe the questions of motion for a gas moving along pipes and arterial fissures and for gas being filtrated through a porous medium, one usually uses empirical laws (D'Arcy's law, the Forkhgeymer relation, etc. [1, 2]).

The system of equations for gasdynamics taking into account resistance forces is given in [3, 4], and an analysis of a system of general, quasilinear equations was conducted in [5].

This study considers the flow of gas which is described by the Euler equation with resistance forces.

The system of equations which describes the flow of isothermal gas has the form

$$\frac{\partial}{\partial t}\rho + r^{-\nu}\frac{\partial}{\partial r}(r^{\nu}\rho u) = 0; \tag{1.1}$$

$$\frac{\partial}{\partial t}j + r^{-\nu}\frac{\partial}{\partial r}(r^{\nu}ju) = -\frac{\partial p}{\partial r} - F; \tag{1.2}$$

$$p = c^2\rho, \tag{1.3}$$

where p is the pressure; ρ is the density; u is the velocity of the flow; $j = u\rho$ is the density of the gas flow; c is the isothermal velocity of sound; F is the force of resistance to the gas flow; ν is a symmetry index (ν = 0 for the two-dimensional problem, ν = 1 for the cylindrical problem, and ν = 2 for the spherically symmetric problem); r is the position; and t is the time.

For small Reynold's numbers ($Re = \lambda\rho u m \mu^{-1} \leq 1$)

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$$F = F_1 = \mu m k^{-1} u, \tag{1.4}$$

where k is the permeability coefficient; m is the porosity of the medium; μ is the viscosity of the gas; and λ is the characteristic pore dimension. For $1 \lesssim \mathrm{Re} \lesssim 10$,

$$F = F_2 = \mu m k^{-1} u + \lambda m^2 k^{-1} \rho u^2. \tag{1.5}$$

For the system of equations (1.1)-(1.3), the boundary conditions and initial conditions are assigned as

$$p(r, 0) = p_0,\ u(r, 0) = 0; \tag{1.6}$$

$$p(0, t) = \chi(t),\ u(0, t) = u_0. \tag{1.7}$$

For supersonic gas flow, two families of characteristics arise at the boundary ($dr_1/dt = u + c$, $dr_2/dt = u - c$) and, therefore, the system of quasilinear, hyperbolic equations (1.1)-(1.3) requires two boundary conditions (1.7) [5, 6]. If the gas flow is subsonic, then only one of the boundary conditions (1.7) is used.

2. For linear dependence of the resistance force on the velocity of the gas flow, problem (1.1)-(1.3), (1.6), (1.7) for $\chi(t) = At$ and $p_0 = 0$ is self-similar [7, 8].

The dimensionless analogs of the pressure $f(\theta)$ and velocity $\varphi(\theta)$ are determined by the equations

$$p = Ac^2 t f(\theta),\ u = c\varphi(\theta),\ \theta = r(ct)^{-1}. \tag{2.1}$$

The system of equations (1.1)-(1.3) and conditions (1.6), (1.7) are written in self-similar variables as

$$f'(\varphi - \theta) + \varphi' f = -f - \nu\theta^{-1} f\varphi; \tag{2.2}$$

$$f' + \varphi' f(\varphi - \theta) = -\sigma\varphi - \nu\theta^{-1} f\varphi^2, \tag{2.3}$$

$$f(\theta \to \infty) = 0,\ \varphi(\theta \to \infty) = 0;$$
$$f(\theta = 0) = 1,\ \varphi(\theta = 0) = \varphi_0, \tag{2.4}$$

where $\sigma = \mu m(kA)^{-1}$ and $\varphi_0 = u_0 c^{-1}$.

For filtration flow which obeys the linear D'Arcy's law, we obtain from Eq. (2.3)

$$f' + \sigma\varphi = 0. \tag{2.5}$$

The solution of the system of equations (2.2) and (2.5) for a given pressure at the boundary $f(\theta = 0) = 1$ and for two-dimensional flow ($\nu = 0$) is a simple wave [9]

$$f(\theta) = \begin{cases} 1 - \sigma^{1/2}\theta, & \theta \leqslant \theta_0, \quad \theta_0 = \sigma^{1/2}, \\ 0, & \theta > \theta_0, \end{cases}$$
$$\varphi(\theta) = \begin{cases} \sigma^{-1/2}, & \theta \leqslant \theta_0, \\ 0, & \theta > \theta_0. \end{cases} \tag{2.6}$$

It follows from (2.6) that the velocity of the gas flow is determined by a dimensionless parameter, the coefficient of resistance σ, and is of constant quantity.

For a force of zero resistance ($\sigma = 0$) there is also an analytical solution to the gas-dynamical system of equations (2.2) and (2.3) with the conditions (2.4) in the two-dimensional case ($\nu = 0$): for $\varphi_0 > \sqrt{2}$,

$$(2\sqrt{2})^{-1} \ln\left[\frac{\varphi + \theta - \sqrt{2}}{\varphi_0 - \sqrt{2}}\,\frac{\varphi_0 + \sqrt{2}}{\varphi + \theta + \sqrt{2}}\right] + \varphi - \varphi_0 = -2\theta,$$
$$f = \left[\frac{(\varphi - \theta)^2 - 2}{\varphi_0^2 - 2}\right]^{1/2};$$

for $\varphi_0 = \sqrt{2}$,

$$\varphi = \sqrt{2} + \theta,\ f = \exp\{-\sqrt{2}\theta\};$$

and for $1 \leqq \varphi_0 < 2$,

$$(2\sqrt{2})^{-1}\ln\left[\frac{\sqrt{2}-\varphi-\theta}{\sqrt{2}-\varphi_0}\,\frac{\sqrt{2}+\varphi_0}{\sqrt{2}+\varphi+\theta}\right]+\varphi-\varphi_0=-2\theta, \tag{2.7}$$

$$f=\left[\frac{2-(\varphi+\theta)^2}{2-\varphi_0^2}\right]^{1/2}.$$

It follows from the solutions of (2.7) that for $\varphi_0 > \sqrt{2}\varphi'' > 0$, and for $1 < \varphi_0 < \sqrt{2}$, $\varphi'' < 0$. The qualitative difference in the behavior of the solutions is related to the time dependence of the gas pressure at the boundary $\chi(t) = At$. For $\chi(t) = At^{\alpha}$, $\varphi_0 \neq \sqrt{2}$ and depends on α.

Solutions (2.7) for $f(\theta)$ and $\varphi(\theta)$ are valid only for supersonic inflow of the gas ($\varphi_0 > 1$). For $0 < \varphi_0 < 1$, the integral curves of (2.7) do not have any physical meaning since $f(\theta \to \infty) \neq 0$.

For $\sigma = 0$ and $\varphi_0 > 1$, as $\theta \to \infty$, $\varphi \to \infty$, i.e., the propagation velocity of the gas front is infinite, which corresponds to instantaneous filling of half-space $r > 0$.

3. We will introduce dimensionless variables and parameters with the equations

$$r'=rR^{-1},\quad t'=tcR^{-1},\quad u'=uc^{-1},\quad p'=pp_1^{-1},\quad j'=jcp_1^{-1}, \tag{3.1}$$

where R is the characteristic length over which the motion of the gas is considered.

From now on we will omit the primes, assuming that $r = r'$, $t = t'$, $p = p'$, etc.

In dimensionless variables (3.1), the system of equations (1.1)-(1.3) and conditions (1.6) and (1.7) can be written as

$$\frac{\partial}{\partial t}p+r^{-\nu}\frac{\partial}{\partial r}(r^{\nu}pu)=0; \tag{3.2}$$

$$\frac{\partial}{\partial t}j+r^{-\nu}\frac{\partial}{\partial r}(r^{\nu}ju)+\frac{\partial p}{\partial r}+F=0,\quad j=pu; \tag{3.3}$$

$$F=\sigma_1 u+\sigma_2\rho u^2; \tag{3.4}$$

$$p(r,0)=p_0p_1^{-1}=p_f,\quad u(r,0)=0; \tag{3.5}$$

$$p(0,t)=\chi(t)p_1^{-1},\qquad u(0,t)=u_0c^{-1}=v_0, \tag{3.6}$$

where $\sigma_1 = \mu mR(kp_1)^{-1}$, $\sigma_2 = \lambda m^2cRk^{-1}$.

The solution of the system of equations (3.2)-(3.4) with initial conditions and boundary conditions (3.5)-(3.6) was determined numerically by decomposing the problem into two fractional steps in time [10]. The decomposed system of equations has the form for $t_n \leqq t < t_n + \tau/2$.

$$\frac{1}{2}\frac{\partial}{\partial t}j+r^{-\nu}\frac{\partial}{\partial r}(r^{\nu}p)=\frac{\nu}{r}p-F,\quad \frac{1}{2}\frac{\partial}{\partial t}p=0, \tag{3.7}$$

$$t_n+\tau/2\leqslant t<t_{n+1};$$

$$\frac{1}{2}\frac{\partial}{\partial t}p+r^{-\nu}\frac{\partial}{\partial r}(r^{\nu}pu)=0,\quad \frac{1}{2}\frac{\partial}{\partial t}j+r^{-\nu}\frac{\partial}{\partial r}(r^{\nu}ju)=0, \tag{3.8}$$

where τ is a step in time; $t_{n+1} = t_n + \tau$; $n = 1, 2, \ldots$

The differential operators in (3.7) and (3.8) can be replaced by difference operators. For the first fractional step, then, this system of equations is approximated by the implicit difference scheme on a four-point grid. Calculation of the flow at this step was done by considering the effects of the pressure gradient and the forces of friction. The obtained results were used to calculate p and j for the second fractional step. The transport equations for pressure and flow (3.8) were solved numerically by the method of flux corrections [11, 12]. Such a solution was obtained in two stages. For the first stage, diffusion was included in the difference scheme, which is analogous to artificial viscosity in conventional difference schemes. The quantity of diffusion coefficients was determined by the condi-

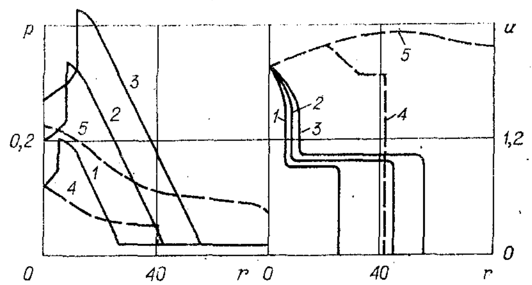

Fig. 1

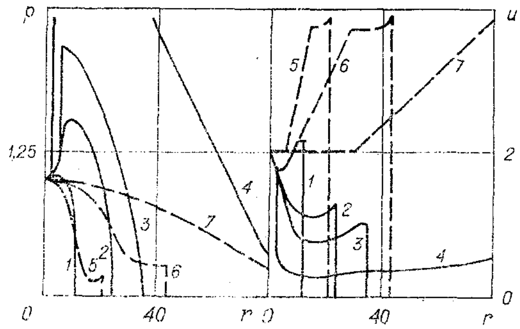

Fig. 2

tions of conservativeness for the difference scheme and by the positivity of the numerical solution. Hence, the difference scheme became monotonic. The second stage involved corrections of the obtained solution by introducing antidiffusion. The errors which entered into the first stage due to diffusion then significantly decreased.

There are several modified versions of the method of flux corrections [11, 12]. For optimizing the correlation of this method with a system of transport equations for describing gas flow with shock waves, the transport calculation for the first stage was done using an explicit difference scheme, and, for the second stage, it was done using an implicit scheme.

Using such an algorithm allows one to achieve stability and high accuracy for the solutions, even for intense discontinuities. As was shown in experiments, discontinuities in the solutions described by the system of equations (3.2)-(3.4) only spread to two to three points. Compared to traditional difference schemes, solving the transport equations by the method of flux corrections substantially (three to four times) reduces dispersion errors, errors in amplitude, and oscillations near the solution discontinuities.

The number of steps in position for numerically integrating the system of equations (3.7) and (3.8) was equal to 500. The number of steps in time was determined by the conditions of Courant.

An estimation of the accuracy of the numerical solutions was done using Richardson's method, where solutions were compared for two different steps in position and time, and the maximum error in the solutions was found to be less than 1.5%.

The difference scheme used allowed us to obtain all the unseparated surfaces of discontinuity.

4. As a result of solving the system of equations (3.2)-(3.4), with conditions (3.5) and (3.6), the space–time dependences of the pressure and velocity characteristics of the gas were obtained.

In Fig. 1, the dependences of pressure and velocity on position for a fixed moment in time are given, where $\chi(t)p_1^{-1} = p_f + At$ ($p_f = 0.01$, $A = 0.1$). The boundary value for the velocity of gas inflow was taken as $v_0 = 2$. Curves 1-3 correspond to gas characteristics where the resistance force depends linearly on the velocity ($\sigma_1 = 0.2$), at times t = 1.1, 1.8, and 2.4; curves 4 and 5 show the pressure and velocity without a resistance force ($\sigma_1 = 0$) at times t = 1.0 and 2.1.

The calculation results are similar to the analytical dependences obtained in Sec. 2 for the gas pressure and velocity. Some of the deviations are due to consideration of the effect of background pressure for a given method of calculation. For certain conditions, a shock wave already forms at the initial moment in time and is directed oppositely to the flow of the gas. This is also true for the solution of the self-similar problem (2.2)-(2.4). The gas pressure at the shock wave front increases suddenly, and the gas velocity decreases from supersonic to subsonic. The intensity of the shock wave increases over time, and the sudden pressure change is gradually displaced in the direction of the gas flow. A shock wave is not formed for all supersonic gas inflow velocities but is formed only for some large critical velocities $v_0 > v^* > 1$. For $v_0 = v^*$, the sudden change in gas pressure is located at the boundary. The intensity of the shock wave is related to the value of the dimensionless coefficient of the resistance force σ_1. For an increase in σ_1, the critical value of the velocity v^* also becomes greater.

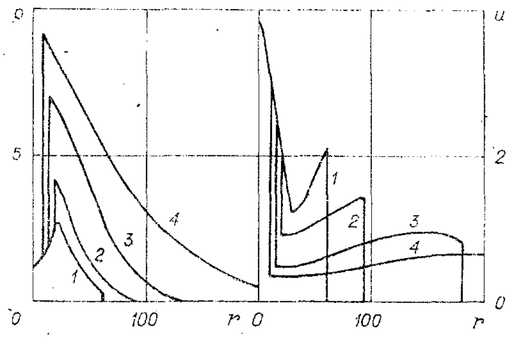

Fig. 3

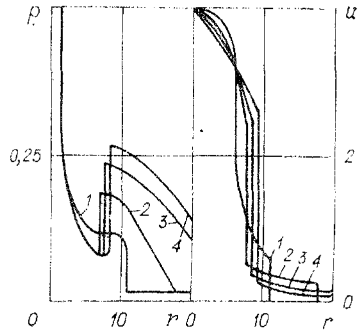

Fig. 4

The dependences of gas pressure (to the left) and velocity (to the right) on position for different moments in time are given in Fig. 2. The boundary condition in this series of calculations was $\chi(t)p_1^{-1}$, the initial pressure was $p_f = 0.01$, and the velocity of gas inflow was $v_0 = 2$. Curves 1-4 correspond to gas flow with a resistance force for t = 0.4, 1.2, 2.5, and 8.5 where $\sigma_1 = 1$; curves 5-7 show dependences of pressure and velocity at times t = 1.3, 3.5, and 7.0 for $\sigma_1 = 0$. For a given boundary condition, the shock wave is not formed at the initial moment in time, but later, $t = t^* \sim 2$, after the motion begins. In addition, the shock wave moves farther toward the boundary with time, i.e., in a direction opposite to the motion of the gas. The intensity of the shock wave increases with time. It is evident in Fig. 2 that for gas flow with a resistance force, there is not only a reflected shock wave, but there is also a direct wave which is propagated in the unperturbed gas. The intensity of the direct wave decreases with time.

The results discussed above correspond to two-dimensional gas flow where the resistance force depends linearly on velocity. To examine the possibility of forming a shock wave using a different dependence F(u), calculations were performed where the resistance force depends on the square of the velocity [Eq. (3.4)].

The boundary conditions were taken to be: $\chi(t)p_1^{-1} = 1$, $v_0 = 4$ for an initial pressure of $p_f = 0.01$. In addition, $\sigma_1 = 10^{-3}$ and $\sigma_2 = 1$.

In Fig. 3, the dependences of pressure and velocity on position for times t = 0.9, 1.8, 5.0, and 10.4 (curves 1-4) are given. For a dependence such as F(u), a shock wave is also formed.

For spherically symmetric flow ($\nu = 2$) at $r = r_0 = 0.1$, the boundary conditions are $p(r_0, t) = 1$, $u(r_0, t) = 4$. The initial pressure is $p_f = 0.01$.

The dependences of gas pressure and flow velocity on position for t = 0.25, 1.0, 5.0, and 9.5 (curves 1-4) when the resistance force depends linearly on velocity ($\sigma_1 = 1$) are shown in Fig. 4.

For spherically symmetric gas flow, in contrast to two-dimensional flow, the pressure decreases with distance from $r = r_0$ to $r = r^* \sim 8$. Beginning with $r > r^*$, the gas pressure increases, and a shock wave forms at some moment in time $t = t^* \sim 0.5$.

We will briefly summarize the results of this study. The solutions of the system of equations (1.1)-(1.3) for some conditions asymptotically coincide with the solutions of the gas filtration equations. However, for gas inflow velocities greater than the speed of sound, a reflected shock wave can form in the medium. Formation of the shock wave can occur both at the initial moment of time and at some later time, after the flow begins. The origin of reflected shock waves during isothermal gas flow with resistance forces is consistent, independent of the geometry of the problem.

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INFLUENCE OF THE SHOCK LAYER ON THE VISCOUS DRAG OF STAR-SHAPED BODIES WITH PLANAR SIDE PANELS

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The ratio of the wave drag determined by the intensity of the corresponding shock layer and the viscous drag due to surface friction is practically clear for three-dimensional bodies of concave cross section, but for star-shaped configurations some further development is necessary. Calculations using linear theory [1], from which we see that the wave drag of a star-shaped body is less than that of a body of revolution of equivalent length and volume, only serve to stress the desirability of such a study.

For a fixed length of a configuration with planar side panels the wave drag is determined by the relative thickness, and depends slightly on the number of petals [2, 3]. As the thickness decreases, for unchanged body length and number of petals, and for a fixed volume, the wave drag decreases, the viscous drag increases, and the size of the petals increases, leading to a considerable increase of the washed surface area [4]. The external inviscid flow for the boundary layer on the configuration surface is the flow behind the bow shock wave (the shock layer). In calculating the viscous drag coefficient one must consider each petal as a flat plate with a skewed leading edge [2, 5, 6]. The friction drag is determined by integrating the local coefficient over the surface.

In this paper we investigate the influence of the shock layer on the boundary layer characteristics and the friction drag. We calculate the friction coefficient as a function of the incident stream parameters and the shape geometry. The wave and viscous drag coefficients are compared.

1. We consider supersonic flow over a star-shaped body, with uniform flow over the planar side panels. The unperturbed flow velocity U_∞ is directed along the body axis. The shape geometry is fully determined by giving the linear size D (the diameter of the cone of equivalent length and volume) and three dimensionless parameters: λ, the elongation (ratio of the

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